Appendix A : Gaussian Distribution Function

- Important because it appears everywhere!
  [e.g. see classwork on spatial distribution of gas atoms]
  [e.g. exam scores among students]
- It is simple : needs only 2 parameters to completely define the distribution†
- One of the three distribution functions in statistics textbooks: binomial, Poisson, normal (gaussian).
- <u>What is it?</u>
  - Variable x is realized randomly with some probability
  - Probability of observing a value between x and $x+dx$ is given by

$$P(x)\,dx = \underbrace{\overbrace{\frac{1}{\sqrt{2\pi\sigma^2}}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}}^{\text{pdf of } x \text{ (pdf = probability distribution function)}}}_{\text{Gaussian (or normal) distribution function}}\, dx$$

  - μ and σ^2 characterize $P(x)$

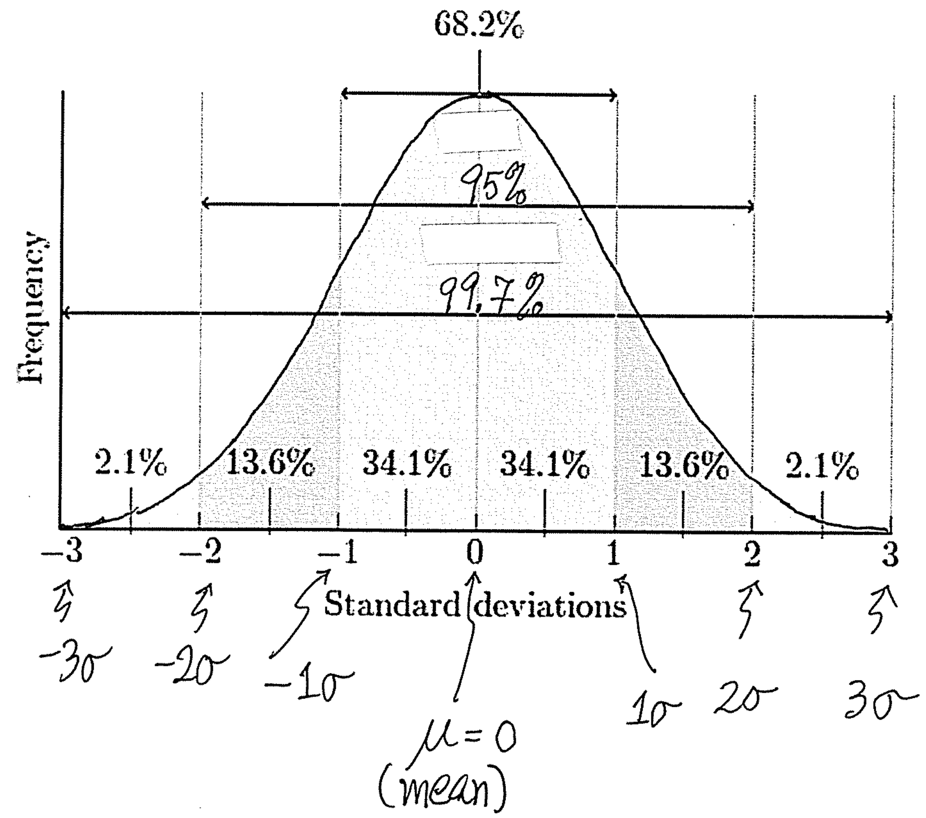
† In contrast, one needs <u>all moments</u> of a general distribution function to completely characterize the distribution.

$$\mu = \text{mean} = \langle x \rangle \equiv \int_{-\infty}^{\infty} dx\, x\, P(x) \qquad 1^{st} \text{ moment}$$

$$\sigma^2 = \text{Variance} = \langle (x-\mu)^2 \rangle \equiv \int_{-\infty}^{\infty} (x-\mu)^2 P(x)$$

σ = standard deviation (SD) (related to 2^{nd} moment)

∴ Gaussian distribution is completely described by its mean and variance.



<u>Recall</u>: Ground state wavefunction of a harmonic oscillator is a gaussian function.